Straight Lines and Pair of Straight Lines

Question1

The point $P(\alpha, \beta)(\alpha > 0, \beta > 0)$ undergoes the following transformations successively.

(a) Translation to a distance of 3 units in positive direction of X-axis.

(b) Reflection about the line $y = -x$.

(c) Rotation of axes through an angle of $\frac{\pi}{4}$ about the origin in the positive direction.

If the final position of that point P is $(-4\sqrt{2}, -2\sqrt{2})$, then $(\alpha + \beta) =$

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

5

B.

7

C.

$6\sqrt{2}$

D.

$2\sqrt{2}$

Answer: A

Solution:

Rotation by angle θ in the positive direction

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

Let $(x_2, y_2) = (-4\sqrt{2}, -2\sqrt{2})$

Point before rotation (x_1, y_1),

$$x_1 = x_2\cos\left(-\frac{\pi}{4}\right) + y_2\sin\left(-\frac{\pi}{4}\right)$$
$$= x_2\left(\frac{\sqrt{2}}{2}\right) - y_2\left(\frac{\sqrt{2}}{2}\right)$$

And $y_1 = x_2\left(\frac{\sqrt{2}}{2}\right) + y_2\left(\frac{\sqrt{2}}{2}\right)$

$$\Rightarrow \quad x_1 = \frac{\sqrt{2}}{2}(-4\sqrt{2}) - \frac{\sqrt{2}}{2}(-2\sqrt{2})$$
$$= -4 + 2 = -2$$
$$\text{And } y_1 = \frac{\sqrt{2}}{2}(-4\sqrt{2}) + \frac{\sqrt{2}}{2}(-2\sqrt{2})$$
$$= -4 - 2 = -6$$

So, point before rotation is $(-2, -6)$ reflection about $y = -x$

$\Rightarrow$ Reflection of point (x, y) is $(-y, -x)$ i.e., $(6, 2)$

Translation by +3 in X-axis

$$\therefore \quad (\alpha, \beta) = (6, -3, 2)$$
$$= (3, 2)$$
$$\therefore \quad \alpha + \beta = 3 + 2 = 5$$

Question2

If the line passing through the point $(4, -3)$ and having negative slope makes an angle of 45° with the line joining the points $(1, 1), (2, 3)$, then the sum of intercepts of that line is

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

$\frac{7}{3}$

B.

1

C.

12

D.

$\frac{26}{3}$

Answer: C

Solution:

Let slope of line passing through the point $(4, -3)$ be m

And slope of the line joining point $(1, 1)$ and $(2, 3)$ is $= \frac{3-1}{2-1} = 2$

$$\begin{aligned} \therefore \quad \tan 45^\circ &= \left|\frac{m-2}{1+2m}\right| \\ \Rightarrow \quad 1 &= \left|\frac{m-2}{1+2m}\right| \end{aligned}$$

taking positive sign, we get

$$1 = \frac{m-2}{1+2m}$$
$$\Rightarrow 1 + 2m = m - 2$$
$$\Rightarrow \quad m = -3$$

Taking negative sign, we get

$$-1 = \frac{m-2}{1+2m}$$
$$\Rightarrow \quad m - 2 = -1 - 2m \Rightarrow 3m = 1$$

$$\Rightarrow \quad m = \frac{1}{3}$$

Since, slope is negative.

$\therefore \quad m = -3$

Equation of line is

$$y + 3 = (-3)(x - 4)$$
$$\Rightarrow \quad y + 3 = -3x + 12$$
$$\Rightarrow \quad 3x + y = 9$$

$\therefore$ x-intercept, when $y = 0$, is $3x = 9$

$\Rightarrow \quad x = 3$

and y-intercept, when $x = 0$ is $y = 9$

$\therefore$ Sum of intercepts $= 3 + 9 = 12$

Question3

$O(0,0), B(-3,-1)$ and $C(-1,-3)$ are vertices of a $\triangle OBC$. D is a point on OC and E is a point on OB. If the equation of DE is $2x + 2y + \sqrt{2} = 0$, then the ratio in which the line DE divides the altitude of the $\triangle OBC$ is

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

$\sqrt{2} : 4\sqrt{2} + 2$

B.

$1 : 4\sqrt{2} + 1$

C.

$\sqrt{2} : 4\sqrt{2} - 2$

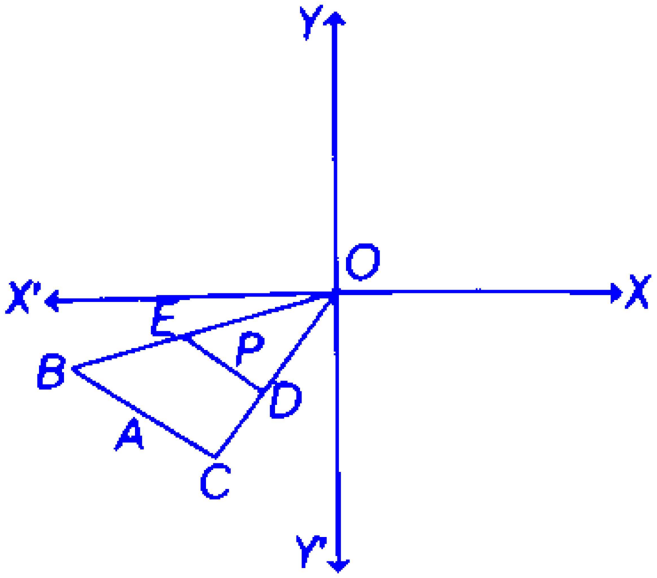
D.

$1 : 4\sqrt{2} - 1$

Answer: D

Solution:

We have $O(0,0), B(-3,-1), C(-1,-3)$ are vertices of a $\triangle OBC$ and equation of DE is $2x + 2y + \sqrt{2} = 0$



Slope of BC is $\frac{-3-(-1)}{-1-(-3)} = \frac{-2}{2} = -1$

$\therefore$ Slope of attitude from 0 to BC is 1 ($\perp$ to BC)

$\therefore$ Equation of altitude with slope passes through $(0, 0)$

$$y - 0 = 1(x - 0)$$
$$\Rightarrow \quad y = x$$

Equation of BC

$y + 1 = (-1)(x + 3) \Rightarrow y = -x - 4$

Point of intersection of line BC and altitude

$$x = -x - 4$$
$$\Rightarrow \quad 2x = -4$$
$$\Rightarrow \quad x = -2$$
$$\therefore \quad y = -2$$

$\therefore$ Foot of perpendicular $A = (-2, -2)$

Now, $2x + 2y + \sqrt{2} = 0$

$$\Rightarrow \quad x + y = -\frac{\sqrt{2}}{2}$$
$$\Rightarrow \quad x + x = \frac{-\sqrt{2}}{2} \quad (\text{ altitude } y = x)$$
$$\Rightarrow \quad 2x = \frac{-\sqrt{2}}{2}$$
$$\Rightarrow \quad x = \frac{-\sqrt{2}}{4}$$

Also, $\quad y = \frac{-\sqrt{2}}{4}$

$\therefore$ Intersection of altitude and DE is P. $\left(\frac{-\sqrt{2}}{4}, \frac{-\sqrt{2}}{4}\right)$

$$\therefore AP = \sqrt{\left(-2+\frac{\sqrt{2}}{4}\right)^2 + \left(-2+\frac{\sqrt{2}}{4}\right)^2}$$

$$= \sqrt{2\left(-2+\frac{\sqrt{2}}{4}\right)^2} = \sqrt{2}\left(-2+\frac{\sqrt{2}}{4}\right)$$

And $PO = \sqrt{2\left(\frac{\sqrt{2}}{4}\right)^2}$

$$= \sqrt{2 \times \frac{2}{16}} = \frac{1}{2}$$

$$\therefore \frac{PO}{AP} = \frac{\frac{1}{2}}{\sqrt{2}\left(-2+\frac{\sqrt{2}}{4}\right)} = 1 : 4\sqrt{2} - 1$$

Question4

Every point on the curve $3x + 2y - 3xy = 0$ is the centroid of a triangle formed by the coordinate axes and a line (L) intersecting both the coordinates axes. Then, all such lines (L)

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

are parallel

B.

are concurrent

C.

intersect each other at different points

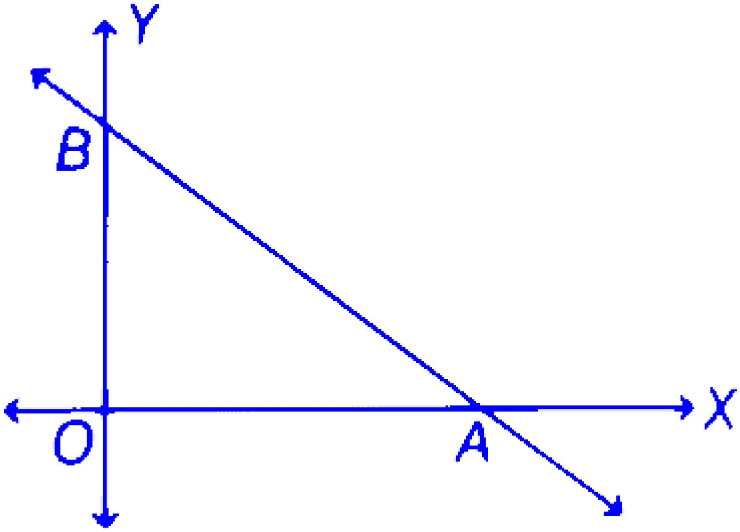
D.

are perpendicular to the tangents to the curve

Answer: B

Solution:

Given equation of curve $3x + 2y - 3xy = 0$ every point on the curve is the centroid of a triangle formed by the coordinate axes and a line (L) Since, L intersect the coordinate axes



$\therefore$ let $A = (a, 0)$
$B = (0, b)$

$\therefore$ Centroid of $\triangle OAB$,

$$(x, y) = \left(\frac{0+0+a}{3}, \frac{0+b+0}{3}\right) = \left(\frac{a}{3}, \frac{b}{3}\right)$$
$$\Rightarrow a = 3x, b = 3y$$

Equation of line

$$\frac{X}{3x} + \frac{Y}{3y} = 1$$
$$\Rightarrow \frac{X}{x} + \frac{Y}{y} = 3 \qquad \ldots(i)$$

Since, centroid (x, y) lies on given curve

$$3x + 2y - 3xy = 0$$
$$\Rightarrow \quad 3x + 2y = 3xy$$
$$\Rightarrow \quad 3xy = 3x + 2y \Rightarrow 3 = \frac{3x}{xy} + \frac{2y}{xy}$$
$$\Rightarrow \quad \frac{X}{x} + \frac{Y}{y} = \frac{3}{y} + \frac{2}{x} \quad \text{(Using Eq. (i))}$$
$$\Rightarrow \quad \frac{X-2}{x} + \frac{Y-3}{y} = 0$$

$\Rightarrow$ Does not represents to a single line

Now, $3x + 2y = 3xy$

$$\Rightarrow \quad \frac{3x}{3xy}+\frac{2y}{3xy}=1$$
$$\Rightarrow \quad \frac{1}{y}+\frac{2}{3x}=1$$
$$\Rightarrow \quad \frac{1}{\frac{b}{3}}+\frac{2}{3\left(\frac{a}{3}\right)}=1$$
$$\Rightarrow \quad \frac{3}{b}+\frac{2}{a}=1 \Rightarrow 3a+2a=ab$$
$$\Rightarrow \quad ab-3a-2b=0$$
$$\Rightarrow \quad a=\frac{2b}{b-3}$$

Consider the equation of line L

$bX+aY=ab$

$$\Rightarrow \quad bX+\frac{2b}{b-3}Y=b\left(\frac{2b}{b-3}\right)$$
$$\Rightarrow \quad X+\frac{2}{b-3}Y=\frac{2b}{b-3}$$
$$\Rightarrow \quad X(b-3)+2Y=2b$$
$$\Rightarrow \quad b(X-2)+(-3X+2Y)=0$$

For all such lines L

$X-2=0 \Rightarrow X=2$

and $\quad -3X+2Y=0$

$\Rightarrow \quad Y=3$

$\Rightarrow \quad L$ is passes through $(2,3)$

$\Rightarrow \quad L$ are concurrent.

Question5

The value of ' a ' for which the equation $\left(a^2-3\right)x^2+16xy-2ay^2+4x-8y-2=0$ represents a pair of perpendicular lines is

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

2

B.

-1

C.

3

D.

4

Answer: C

Solution:

Given, equation

$\left(a^2-3\right) x^2+16 x y-2 a y^2+4 x-8 y-2=0$

represents a pair of perpendicular line

$$\begin{aligned} & \therefore\left(a^2-3\right)+(-2 a)=0 \\ & \Rightarrow \quad a^2-2 a-3=0 \\ & \Rightarrow \quad a^2-3 a+a-3=0 \\ & \Rightarrow \quad a(a-3)+1(a-3)=0 \\ & \Rightarrow \quad(a-3)(a+1)=0 \\ & \Rightarrow \quad a=3,-1 \end{aligned}$$

$\Rightarrow a=3$, since for $a=-1$ lines does not exists.

Question6

If the points $A(2,3), B(3,2)$ form a triangle with a variable point $p\left(t, t^2\right)$, where t is a parameter, then the equation of the locus of the centroid of $\triangle ABC$ is

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

$9 x^2-30 x-3 y+20=0$

B.

$3x^2 - 10x - y + 10 = 0$

C.

$9y^2 - 30y - 3x + 20 = 0$

D.

$3y^2 - 10y - x + 10 = 0$

Answer: B

Solution:

Let the centroid be $G(x, y)$. The coordinates of the centroid of triangles with vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are

$x = \frac{x_1+x_2+x_3}{3}$ and $y = \frac{y_1+y_2+y_3}{3}$

Substituting the coordinates of A, B and P.

$$x = \frac{2+3+t}{3} = \frac{5+t}{3},$$
$$y = \frac{3+2+t^2}{3} = \frac{5+t^2}{3}$$

$$\Rightarrow \quad 3x - 5 = t, y = \frac{5+(3x-5)^2}{3}$$
$$(\because t = 2x - 5)$$
$$\Rightarrow \quad 3y = 5 + 9x^2 + 25 - 30x$$
$$\Rightarrow \quad 9x^2 - 30x - 3y + 30 = 0$$
$$\Rightarrow \quad 3x^2 - 10x - y + 10 = 0$$

The equation of the locus of the centroid of $\triangle ABC$ is $3x^2 - 10x - y + 10 = 0$

Question7

If (h, k) is the new origin to be chosen to eliminate first degree terms from the equation $S \equiv 2x^2 - xy - y^2 - 3x + 3y = 0$ by translation and if θ is the angle with which the axes are to be rotated about the origin in anti-clockwise direction to eliminate xy-term from $S = 0$, then $\tan 2\theta =$

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

$h+k$

B.

$h-k$

C.

hk

D.

$-\frac{h}{3k}$

Answer: D

Solution:

Let $S(x,y)=2x^2-xy-y^2-3x+3y$

$\frac{\partial S}{\partial x}=4x-y-3, \frac{\partial S}{\partial y}=-x-2y+3$

Solve, $\frac{\partial S}{\partial x}=0$ and $\frac{\partial S}{\partial y}=0$, we get

$4x-y-3=0$ and $-x-2y+3=0$

Solving these two equations for $x=h$ and $y=k$

$4h-k-3=0$ and $-h-2k+3=0$
$\therefore h=1, k=1$

So, the new origin is $(h,k)=(1,1)$

We know that a general second-degree equation

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$, the angle of rotation θ satisfies

$\tan 2\theta=\frac{B}{A-C}$

Here, $S\equiv 2x^2-xy-y^2-3x+3y=0$, we have

$A=2B=-1$ and $C=-1$

$$\therefore \tan 2\theta=\frac{-1}{2-(-1)}=-\frac{1}{3}$$

Now, $h+k=1+1=2h-k=1-1=0,$

$hk=1\times1=1, \frac{h}{3k}=\frac{1}{3}$

So, $\tan 2\theta=\frac{-1}{3}=\frac{-h}{3k}$

Question8

A line L perpendicular to the line $5x-12y+6=0$ makes positive intercept on the Y-axis. If the distance from the origin to the line L is 2 units and the angle made by the perpendicular drawn from the origin to the line L with positive X-axis is θ, then $\tan\theta+\cot\theta=$

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

$\frac{25}{12}$

B.

$\frac{625}{168}$

C.

$\frac{169}{60}$

D.

$\frac{1681}{360}$

Answer: C

Solution:

Give line is $5x-12y+6=0$

Slope of this line is $m_1=\frac{-5}{-12}=\frac{5}{12}$

Since, line L is perpendicular to this line, the product of their slope is -1 . Let the slope of line L be m_L.

Then, $m_L\times\frac{5}{12}=-1$

$\Rightarrow \quad m_L=-\frac{12}{5}$

The equation of a line in normal form is $x \cos \theta + y \sin \theta = P$, where P is the distance from the origin to the line and θ is the angle made by the perpendicular from the origin.

We have $P = 2$ unit, So, the equation of line is $x \cos \theta + y \sin \theta = 2$

Slope, $m_L = \frac{-\cos \theta}{\sin \theta} = -\cot \theta$

$\therefore -\cot \theta = \frac{-12}{5} \Rightarrow \cot \theta = \frac{12}{5}$

For y-intercept, $x = 0$ then $y \sin \theta = 2$

$\Rightarrow \quad y = \frac{2}{\sin \theta}$

And since y-intercept is positive, so

$$\frac{2}{\sin \theta} > 0$$
$$\Rightarrow \sin \theta > 0$$

$\Rightarrow \cot \theta = \frac{12}{5} > 0, \theta$ must be in first quadrant.

Now, $\tan \theta = \frac{1}{\cot \theta} = \frac{5}{12}$

So, $\tan \theta + \cot \theta = \frac{5}{12} + \frac{12}{5}$

$= \frac{25+144}{60} = \frac{169}{60}$

Question9

If a line L passing through a point $A(2, 3)$ intersects another line $4x - 3y - 19 = 0$ at the point B such that $AB = 4$, then the angle made by the line L with positive X-axis in anti-clockwise direction is

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

$\tan^{-1}\left(-\frac{3}{4}\right)$

B.

$\tan^{-1}\left(\frac{3}{4}\right)$

C.

$\frac{\pi}{4}$

D.

$-\frac{\pi}{4}$

Answer: A

Solution:

Let the angle made by Line L with positive X-axis be θ.

So, $B=(Ax+AB\cos\theta, Ay+AB\sin\theta)$

Given, $A(2,3)$ and $AB=4$, we have

$B=(2+4\cos\theta, 3+4\sin\theta)$

Since, point B lies on the line $4x-3y-19=0$, substitute the coordinate of B into this equation.

$$\begin{aligned}
&4(2+4\cos\theta)-3(3+4\sin\theta)-19=0\\
\Rightarrow\quad &8+16\cos\theta-9-12\sin\theta-19=0\\
\Rightarrow\quad &16\cos\theta-12\sin\theta-20=0\\
\Rightarrow\quad &4\cos\theta-3\sin\theta-5=0\\
\Rightarrow\quad &\frac{4}{5}\cos\theta-\frac{3}{5}\sin\theta=1
\end{aligned}$$

Let $\cos\alpha=\frac{4}{5}$ and $\sin\alpha=\frac{3}{5}$ for some angle α in the first quadrant.

$\therefore \cos\alpha\cdot\cos\theta-\sin\alpha\cdot\sin\theta=1$

$$\begin{aligned}
\Rightarrow\quad &\cos(\alpha+\theta)=1\\
\Rightarrow\quad &\alpha+\theta=2n\pi,\ \text{for some integer } n.\\
\Rightarrow\quad &\alpha+\theta=0\\
\Rightarrow\quad &\alpha=-\theta \text{ or } \alpha+\theta=2\pi\\
\Rightarrow\quad &\theta=2\pi-\alpha
\end{aligned}$$

The angle made by the line L with positive X-axis in the anti-clockwise direction is taken as a positive angle, where

$\cos\theta=\frac{4}{5}$ and $\sin\theta=-\frac{3}{5}$

This angle lies in the fourth quadrant.

So, $\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{-3}{4}$

$\Rightarrow\quad \theta=\tan^{-1}\left(\frac{-3}{4}\right)=-\tan^{-1}\left(\frac{3}{4}\right)$

Question10

A variable straight-line L with negative slope passes through the point $(4, 9)$ and cuts the positive coordinate axes in A and B. If O is the origin, then the minimum value of $OA + OB$ is

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

25

B.

12

C.

13

D.

5

Answer: A

Solution:

Let the equation of the line L be $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are the x and y intercepts, respectively. So, $OA = a$ and $OB = b, a > 0, b > 0$.

Here, the line passes through the point $(4, 9)$, So,

$\frac{4}{a} + \frac{9}{b} = 1$

We want to minimize $a + b$. So consider

$$\begin{aligned}(a+b)\left(\frac{4}{a}+\frac{9}{b}\right) &= 4+\frac{9a}{b}+\frac{4b}{a}+9 \\ &= 13+\frac{9a}{b}+\frac{4b}{a}\end{aligned}$$

By AM-GM inequality, for positive numbers,

$$\frac{9a}{b}+\frac{4b}{a} \geq \sqrt[2]{\frac{9a}{b}\cdot\frac{4b}{a}}$$
$$\Rightarrow \quad 2\sqrt{36}=2\cdot 6=12$$

So, $13+\frac{9a}{b}+\frac{4b}{a}=13+12=25$

But, since $(a+b)\left(\frac{4}{a}+\frac{9}{b}\right)=(a+b)\cdot 1$

$=a+b \quad$ (Using Eq. (i))

$\therefore$ The minimum value of $a+b$ is 25 and this occurs when $\frac{9a}{b}=\frac{4b}{a}$

$$\Rightarrow \quad 9a^2=4b^2 \text{ or } 3a=2b \quad (\because a,b>0)$$
$$\Rightarrow \quad b=\frac{3}{2}a$$

So, $\frac{4}{a}+\frac{9}{b}=1$

$$\Rightarrow \frac{4}{a}+\frac{9}{\frac{3}{2}a}=1 \Rightarrow \frac{4}{a}+\frac{18}{3a}=1$$
$$\Rightarrow \frac{4}{a}+\frac{6}{a}=1 \Rightarrow \frac{10}{a}=1$$
$$\Rightarrow \quad a=10$$

Then, $b=\frac{3}{2}a=\frac{3}{2}\times 10=15$

So, the minimum value of $OA+OB$ is $a+b=10+15=25$

Question11

If $4x^2+12xy+9y^2+2gx+2fy-1=0$ represent a pair of parallel lines, then

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

$\frac{f}{g}+\frac{g}{f}+\frac{13}{6}=0$

B.

$f^2+g^2=fg$

C.

$f^2+g^2=6fg$

D.

$\frac{f}{g}+\frac{g}{f}=\frac{13}{6}$

Answer: D

Solution:

Given equation,

$4x^2+12xy+9y^2+2gx+2fy-1=0$

Since, general second-degree equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of straight lines, if

$\Delta=abc+2fgh-bg^2-ch^2-af^2=0$

Here, $a=4, b=9, h=6$

So, $\Delta=4\times 9\times c+2fg\times 6-9g^2$

$$\begin{aligned} & -c(6)^2-4f^2=0 \\ & =36c+12fg-9g^2-36c-4f^2=0 \\ & =12fg-9g^2-4f^2=0 \\ \Rightarrow \quad & -(3g-2f)^2=0 \Rightarrow 3g-2f=0 \\ \Rightarrow & \frac{g}{f}=\frac{2}{3} \text{ and } \frac{f}{g}=\frac{3}{2} \end{aligned}$$

So, $\frac{f}{g}+\frac{g}{f}=\frac{3}{2}+\frac{2}{3}=\frac{9+4}{6}=\frac{13}{6}$

Question12

(a, b) is the point to which the origin has to be shifted by translation of axes so as to remove the first-degree terms from the equation $2x^2-3xy+4y^2+5y-6=0$. If the angle by which the axes are to be rotated in positive direction about the origin to remove the xy-term from the equation $ax^2+23abxy+by^2=0$ is θ, then $\tan 2\theta=$

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $\frac{\pi}{4}$

B. 60

C. $\frac{\pi}{3}$

D. 15

Answer: B

Solution:

To solve the problem of shifting the origin to the point (a, b) in order to eliminate the first-degree terms from the equation $2x^2 - 3xy + 4y^2 + 5y - 6 = 0$, we start by considering new coordinates $(x + a, y + b)$.

Substituting these into the equation gives:

$$2(x+a)^2 - 3(x+a)(y+b) + 4(y+b)^2 + 5(y+b) - 6 = 0$$

Expanding this, we get:

$$2(x^2 + 2ax + a^2) - 3(xy + ay + bx + ab) + 4(y^2 + 2by + b^2) \\ +5y + 5b - 6 = 0$$

Simplifying further, the remaining first-degree terms become:

$$(4a - 3b)x - (3a - 8b - 5)y = 0$$

For these terms to be eliminated, we require:

$$4a - 3b = 0 \quad \text{and} \quad 3a - 8b - 5 = 0$$

Solving these equations simultaneously, we find:

$$a = -\frac{15}{23}, \quad b = \frac{-20}{23}$$

Now, consider the transformed equation $ax^2 + 23abxy + by^2 = 0$. We need to find the angle θ to rotate the axes such that the xy-term disappears. This involves calculating $\tan 2\theta$, given by:

$$\tan 2\theta = \frac{23ab}{a-b}$$

Substituting the values of a and b:

$$\tan 2\theta = \frac{23 \times \frac{-15}{23} \times \frac{-20}{23}}{-\frac{15}{23} + \frac{20}{23}} = 60$$

Therefore, $\tan 2\theta = 60$.

Question13

$A(1, -2), B(-2, 3), C(-1, -3)$ are the vertices of a $\triangle ABC$. L_1 is the perpendicular drawn from A to BC and L_2 is the perpendicular bisector of AB. If (l, m) is the point of intersection of L_1 and L_2,

then $26m - 3 =$

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A.

261

B.

89

C.

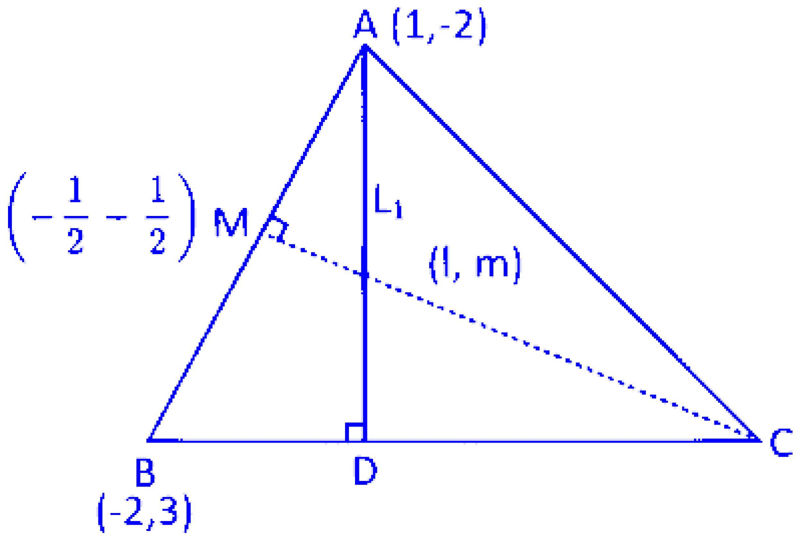
13

D.

431

Answer: B

Solution:



Slope of $BC = \frac{-6}{1} = -6$

Equation of L_1

$$y + 2 = \frac{1}{6}(x - 1)$$

$\Rightarrow \quad 6y + 12 = x - 1$

$$6y = x - 13$$

Slope of $AB = \frac{5}{-3}$

Equation of L_2

$y - \frac{1}{2} = \frac{3}{5}\left(x + \frac{1}{2}\right) = \frac{3x}{5} + \frac{3}{10}$

$y = \frac{3x}{5} + \frac{3}{10} + \frac{1}{2} = \frac{3x}{5} + \frac{4}{5} \cdots$

On solving Eqs. (i) and (ii), we get

$$6\left(\frac{3x}{5} + \frac{4}{5}\right) = x - 13$$

$$\Rightarrow \quad 18x + 24 = 5x - 65$$

$$\Rightarrow \quad 13x = -89$$

$$\Rightarrow \quad l = -\frac{89}{13}$$

and $6m = \frac{-89}{13} - 13$

$= -\left(\frac{89+169}{13}\right) = -\frac{258}{13}$

$\therefore \quad m = -\frac{43}{13}$

$(l, m) = \left(\frac{-89}{13}, \frac{-43}{13}\right)$

Hence, $26m - 3 = 26\left(\frac{-43}{13}\right) - 3$

$= -86 - 3 = -89$

Question14

The area of the parallelogram formed by the lines $L_1 \equiv \lambda x + 4y + 2 = 0, L_2 \equiv 3x + 4y - 3 = 0,$ $L_3 \equiv 2x + \mu y + 6 = 0, L_4 \equiv 2x + y + 3 = 0,$ where L_1 is parallel to L_2 and L_3 is parallel to L_4 is

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. 9

B. 7

C. 5

D. 3

Answer: D

Solution:

To find the area of the parallelogram formed by the given lines, we start by understanding the relationships between the lines. We have the lines:

$L_1 : \lambda x + 4y + 2 = 0$

$L_2 : 3x + 4y - 3 = 0$

Since L_1 is parallel to L_2, their slopes must be equal. Adjusting for slope, this gives us $\lambda = 3$.

From the equation of $L_2 : 3x + 4y - 3 = 0$, we express y in terms of x:

$4y = -3x + 3 \quad \Rightarrow \quad y = -\frac{3}{4}x + \frac{3}{4}$

So, the slope of L_2 (and therefore L_1) is $m_1 = -\frac{3}{4}$. The y-intercepts of the parallel lines, L_1 and L_2, are $c_1 = \frac{3}{4}$ and $c_2 = -\frac{2}{4}$ respectively.

Next, consider the lines:

$L_3 : 2x + \mu y + 6 = 0$

$L_4 : 2x + y + 3 = 0$

Given these lines are parallel, solve for the slopes, setting $\mu = 1$ to obtain:

From $L_4 : 2x + y + 3 = 0$:

$y = -2x - 3$

This gives $m_2 = -2$. The y-intercepts are $d_1 = -3$ for L_4 and $d_2 = -6$ for L_3.

Now, calculate the expressions needed for the area of the parallelogram.

The difference between y-intercepts of L_1 and L_2:

$C_1 - C_2 = \frac{3}{4} + \frac{2}{4} = \frac{5}{4}$

The difference between y-intercepts of L_3 and L_4:

$d_1 - d_2 = -3 + 6 = 3$

Finally, calculate the area of the parallelogram using the formula:

$\text{Area} = \frac{(C_1 - C_2) \times (d_1 - d_2)}{|m_1 - m_2|}$

Substituting the values:

$\text{Area} = \frac{\frac{5}{4} \times 3}{|-\frac{3}{4} + 2|} = \frac{15}{4} \times \frac{4}{5} = 3$

Therefore, the area of the parallelogram is 3.

Question15

If the angle between the pair of lines given by the equation $ax^2 + 4xy + 2y^2 = 0$ is $45°$, then the possible values of a

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. are -3 or 21

B. are $-6 \pm 4\sqrt{3}$

C. are $-6 \pm 24\sqrt{2}$

D. do not exist

Answer: B

Solution:

To find the angle between the pair of lines given by the equation $ax^2 + 4xy + 2y^2 = 0$ and knowing that this angle is $45°$, we start by using the formula for the angle θ between two lines given by:

$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$

For this equation, we identify the coefficients:

$h = 2$

$b = 2$

Plugging these into the formula for $\tan\theta$ and setting $\tan\theta = 1$ (since the angle is $45°$):

$\frac{2\sqrt{4-2a}}{a+2} = 1$

Squaring both sides and rearranging gives:

$4(4 - 2a) = (a + 2)^2$

Simplifying further:

$16 - 8a = a^2 + 4 + 4a$

Rearranging terms forms the quadratic equation:

$a^2 + 12a - 12 = 0$

To solve for a, use the quadratic formula $a = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$:

$a = \frac{-12 \pm \sqrt{144+48}}{2} = \frac{-12 \pm \sqrt{192}}{2}$

Since $\sqrt{192} = 8\sqrt{3}$, the solutions for a are:

$a = \frac{-12 \pm 8\sqrt{3}}{2} = -6 \pm 4\sqrt{3}$

Therefore, the possible values of a are $-6 \pm 4\sqrt{3}$.

Question16

By shifting the origin to the point $(h, 5)$ by the translation of coordinate axes, if the equation $y = x^3 - 9x^2 + cx - d$ transforms to $Y = X^3$, then $\left(d - \frac{c}{h}\right) =$

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. 0

B. 13

C. 11

D. 25

Answer: B

Solution:

To transform the equation $y = x^3 - 9x^2 + cx - d$ to $Y = X^3$ by shifting the origin to the point $(h, 5)$, we use the translation:

$Y = y - 5$

$X = x - h$

Substituting, the equation becomes:

$y - 5 = (x - h)^3$

Expanding the right side, we have:

$y = x^3 - h^3 - 3hx^2 + 3h^2x + 5$

By comparing coefficients with the original equation $y = x^3 - 9x^2 + cx - d$, we deduce:

$-3h = -9$

$c = 3h^2$

$d = h^3 - 5$

Solving these equations:

From $-3h = -9$, we find $h = 3$.

Substituting $h = 3$ in $c = 3h^2$, we get $c = 3 \times 3^2 = 27$.

Substituting $h = 3$ in $d = h^3 - 5$, we find $d = 3^3 - 5 = 22$.

Finally, we compute:

$d - \frac{c}{h} = 22 - \frac{27}{3} = 22 - 9 = 13$

Question17

The equation of the straight line whose slope is $\frac{-2}{3}$ and which divides the line segment joining $(1, 2)$, $(-3, 5)$ in the ratio $4 : 3$ externally is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $2x + 3y - 12 = 0$

B. $3x + 2y + 27 = 0$

C. $2x + 3y - 9 = 0$

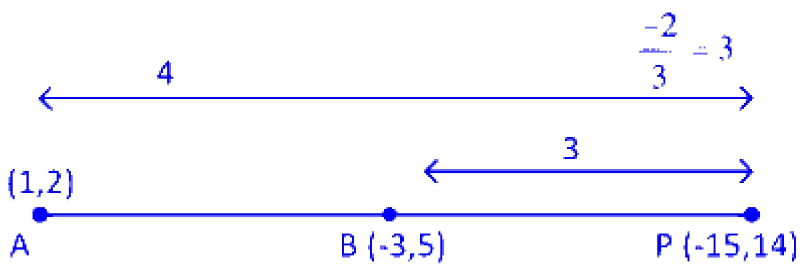
D. $2x + 3y + 12 = 0$

Answer: A

Solution:

Let the point P divide

then, $P \equiv \left(\frac{-12-3}{4-3}, \frac{20-6}{4-3}\right)$



$\Rightarrow$ Slope $= -\frac{2}{3}$

$\Rightarrow$ $\frac{y-14}{x+15} = -\frac{2}{3}$ {Point slope form }

$3y - 42 = -2x - 30$

$2x + 3y - 12 = 0$

Question18

$7x + y - 24 = 0$ and $x + 7y - 24 = 0$ represent the equal sides of an isosceles triangle. If the third side passes through $(-1, 1)$ then, a possible equation for the third side is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $3x - y = -4$

B. $x + y = 0$

C. $x - 2y = -3$

D. $3x + y = -2$

Answer: B

Solution:

Let

$AB : 7x + y - 24 = 0$

Slope $= m_1 = -7$

$AC : x + 7y - 24 = 0,$

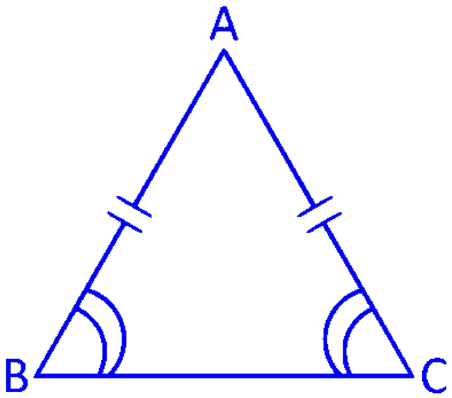
Slope $= m_2 = \frac{-1}{7}$

BC : passes through $(-1, 1)$

Let slope $= m$

then equation of $BC : y - 1 = m(x + 1)$

$\Rightarrow mx - y + m + 1 = 0 \quad \ldots$ (i)



$\therefore AB = AC \Rightarrow \angle ABC = \angle ACB$

$\Rightarrow \left|\frac{m+7}{1-7m}\right| = \left|\frac{m+\frac{1}{7}}{1-\frac{m}{7}}\right|$

$\left(\because \tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|\right)$

$\Rightarrow \left|\frac{m+7}{1-7m}\right| = \left|\frac{7m+1}{7-m}\right|$

$\Rightarrow \left|49 - m^2\right| = \left|49m^2 - 1\right|$

$\Rightarrow 49 - m^2 = \pm\left(49m^2 - 1\right)$

$\Rightarrow \quad m = \pm 1$

So, equation of third line is $x - y + 2 = 0$ or $x + y = 0$

Question19

The combined equation of a possible pair of adjacent sides of a square with area 16 square units whose centre is the point of intresection of the lines $x + 2y - 3 = 0$ and $2x - y - 1 = 0$ is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $(2x - y - 1 + 4\sqrt{5})(x + 2y - 3 + 4\sqrt{5}) = 0$

B. $(2x - y - 1 - 4\sqrt{5})(x + 2y - 4\sqrt{5}) = 0$

C. $(2x - y - 2\sqrt{5})(x + 2y + 2\sqrt{5}) = 0$

D. $(2x - y - 1 - 2\sqrt{5})(x + 2y - 3 + 2\sqrt{5}) = 0$

Answer: D

Solution:

We are given two lines:

$x + 2y - 3 = 0$

$2x - y - 1 = 0$

The intersection point of these lines is $(1, 1)$.

The area of the square is given as 16 square units, which implies the side length a is 4, since:

$a^2 = 16 \quad \Rightarrow \quad a = 4$

The perpendicular distance from the center $(1, 1)$ to each side of the square is half the side length:

Perpendicular distance $= \frac{a}{2} = 2$

Next, we check the perpendicular distance of point $(1, 1)$ from the line $2x - y - 1 = 2\sqrt{5}$:

$$\text{Distance} = \left|\frac{2(1) - 1 - 2\sqrt{5}}{\sqrt{5}}\right| = \left|\frac{2 - 1 - 2\sqrt{5}}{\sqrt{5}}\right| = \left|\frac{1 - 2\sqrt{5}}{\sqrt{5}}\right| = |2| = 2$$

Then, we check the perpendicular distance of point $(1, 1)$ from the line $x + 2y - 3 + 2\sqrt{5} = 0$:

$$\text{Distance} = \left|\frac{1 + 2(1) - 3 + 2\sqrt{5}}{\sqrt{5}}\right| = \left|\frac{3 - 3 + 2\sqrt{5}}{\sqrt{5}}\right| = 2$$

Both distances are indeed equal to 2, confirming the side length of the square. This verifies the equations of the sides for this configuration:

$(2x - y - 1 - 2\sqrt{5})(x + 2y - 3 + 2\sqrt{5}) = 0$

Question20

If the line $2x + by + 5 = 0$ forms an equilateral to triangle with $ax^2 - 96bxy + ky^2 = 0$, then $a + 3k =$

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $3b$

B. 192

C. $4b^2$

D. 102

Answer: B

Solution:

We are given the line equation $2x + by + 5 = 0$ and the quadratic equation $ax^2 - 96bxy + ky^2 = 0$. We need to find the value of $a + 3k$ when these form an equilateral triangle.

The angle between the lines derived from the quadratic equation can be represented as:

$2\left|\frac{\sqrt{h^2-ak}}{a+k}\right| = \tan 60^\circ$

Given that $\tan 60^\circ = \sqrt{3}$, substituting into the equation we get:

$\sqrt{3} = \frac{2\sqrt{(48b)^2-ak}}{a+k}$

Squaring both sides:

$3(a+k)^2 = 4(48b)^2 - 4ak$

Expanding and rearranging terms, we have:

$3a^2 + 3k^2 + 6ak = 4(48)^2b^2 - 4ak$

Adding $4ak$ to both sides:

$3a^2 + 3k^2 + 10ak = 4 \times 48^2b^2$

Hence, factoring gives:

$(a + 3k)(3a + k) = 4 \times 48 \times 48b^2$

Therefore, the possible values for $a + 3k$ could be 192 or $48b^2$. Comparing with the options, it is clear:

$a + 3k = 192$

Question21

If the distance from a variable point P to the point $(4, 3)$ is equal to the perpendicular distance from P to the line $x + 2y - 1 = 0$, then the equation of the locus of the point P

is

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. $4x^2 + 4xy + y^2 - 38x + 26y + 124 = 0$

B. $4x^2 - 4xy + y^2 - 38x - 26y + 124 = 0$

C. $4x^2 - 4xy + y^2 + 38x + 26y + 124 = 0$

D. $4x^2 - 4xy + y^2 - 38x + 26y + 124 = 0$

Answer: B

Solution:

Let the variable point be $P(h, k)$.

The distance from P to the point $(4, 3)$ can be expressed as:

$\sqrt{(h-4)^2 + (k-3)^2}$

The perpendicular distance from P to the line $x + 2y - 1 = 0$ is given by:

$\left|\frac{h+2k-1}{\sqrt{1^2+2^2}}\right| = \left|\frac{h+2k-1}{\sqrt{5}}\right|$

According to the problem, these two distances are equal:

$\sqrt{(h-4)^2 + (k-3)^2} = \left|\frac{h+2k-1}{\sqrt{5}}\right|$

Squaring both sides, we get:

$(h-4)^2 + (k-3)^2 = \frac{(h+2k-1)^2}{5}$

Expanding both sides:

$h^2 - 8h + 16 + k^2 - 6k + 9 = \frac{h^2+4hk+4k^2-2h-4k+1}{5}$

Multiply through by 5 to eliminate the fraction:

$5(h^2 - 8h + 16 + k^2 - 6k + 9) = h^2 + 4hk + 4k^2 - 2h - 4k + 1$

Simplify and collect like terms:

$5h^2 + 5k^2 - 40h - 30k + 125 = h^2 + 4hk + 4k^2 - 2h - 4k + 1$

Rearranging gives:

$4h^2 + k^2 - 4hk - 38h - 26k + 124 = 0$

Finally, replace (h, k) with (x, y):

$4x^2 + y^2 - 4xy - 38x - 26y + 124 = 0$

Question22

$(0, k)$ is the point to which the origin is to be shifted by the translation of the axes so as to remove the first degree terms from the equation $ax^2 - 2xy + by^2 - 2x + 4y + 1 = 0$ and $\frac{1}{2}\tan^{-1}(2)$ is the angle through which the coordinate axes are to be rotated about the origin to remove the xy-term from the given equation, then $a + b =$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. 1

B. -2

C. 3

D. -4

Answer: C

Solution:

Put $x = X$ and $y = Y + k$

Then,

$\Rightarrow aX^2 - 2X(Y+k) + b(Y+k) - 2X$

$+4(Y+k) + 1 = 0$

$\Rightarrow aX^2 - 2XY - 2kX + bY^2 + bk^2$

$+26kY - 2X + 4Y + 4k + 1 = 0$

Now,

$\because$ coefficient of $x = 0$

$\Rightarrow \quad -2k - 2 = 0$

$\therefore \quad k = -1$

$\because$ Coefficient of $y = 0$

$\Rightarrow \quad 2bk + 4 = 0 \Rightarrow -2b + 4 = 0$

$\therefore \quad b = 2$

Now, put

$X = x\cos\theta - y\sin\theta$

and $\quad Y = x\sin\theta + y\cos\theta$

$\Rightarrow a(x\cos\theta - y\sin\theta)^2 - 2(x\cos\theta - y\sin\theta)$

$(x\sin\theta + y\cos\theta - 1)$

$+2(x\sin\theta + y\cos\theta - 1)^2 - 2(x\cos\theta - y\sin\theta)$

$+4(x\sin\theta + y\cos\theta - 1) + 1 = 0$

[Using Eqs. (i) and (ii)]

Now, coefficient of $xy = 0$

$$\Rightarrow -a(2\cos\theta\sin\theta) - 2\left(\cos^2\theta - \sin^2\theta\right)$$

$$+2(2\sin\theta\cos\theta) = 0$$

$\Rightarrow -a\sin 2\theta - 2\cos 2\theta + 2\sin 2\theta = 0 \quad \ldots$ (iii)

Let $\frac{1}{2}\tan^{-1}(2) = \theta$

$\Rightarrow \quad \tan 2\theta = 2$

Now, from Eq. (iii), we get

$\Rightarrow \frac{-a\times2}{\sqrt{5}} - \frac{2\times1}{\sqrt{5}} + \frac{2\times2}{\sqrt{5}} = 0$

$\left[\because \sin 2\theta = \frac{2}{\sqrt{5}} \text{ and } \cos\theta = \frac{1}{\sqrt{5}}\right]$

$\Rightarrow a = 1$

Hence, $a + b = 1 + 2 = 3$

Question23

β is the angle made by the perpendicular drawn from origin to the line $L \equiv x + y - 2 = 0$ with the positive X-axis in the anticlockwise direction. If a is the X-intercept of the line $L = 0$ and p is the perpendicular distance from the origin to the line $L = 0$, then $a \tan\beta + p^2 =$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. 1

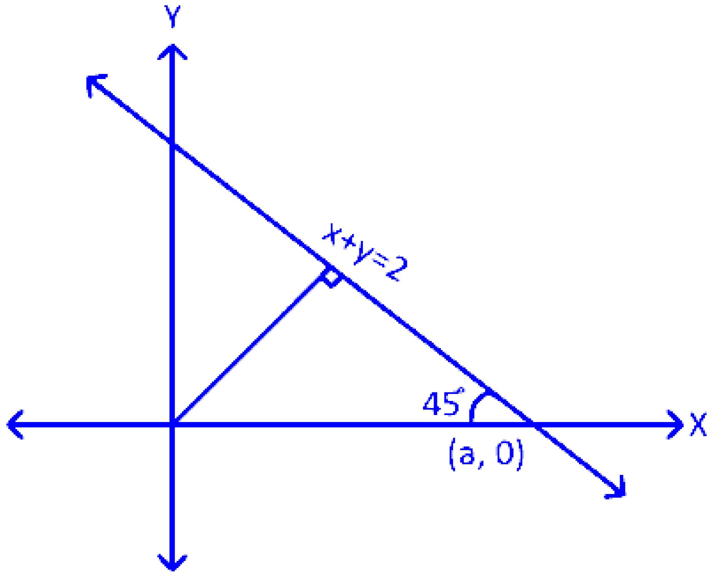
B. 2

C. 3

D. 4

Answer: D

Solution:



For x-intercept, put $y = 0$

$$a = 2$$

$$\therefore \quad P = \left|\frac{0+0-2}{\sqrt{2}}\right| = \sqrt{2}$$

Now; $\tan \beta = \frac{p}{a} = \frac{1}{\sqrt{2}}$

So, $\quad 2 \tan \beta + P^2 \Rightarrow 2 \tan 45^\circ + (\sqrt{2})^2$

$\Rightarrow \quad 2 + 2 = 4$

Question24

The line $2x + y - 3 = 0$ divides the line segment joining the points $A(1, 2)$ and $B(-2, 1)$ in the ratio $a : b$ at the point C. If the point C divides the line segment joining the points $P\left(\frac{b}{3a}, -3\right)$ and $Q\left(-3, -\frac{b}{3a}\right)$ in the ratio $p : q$, then $\frac{p}{q} + \frac{q}{p} =$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. $\frac{29}{10}$

B. $\frac{17}{10}$

C. 6

D. 5

Answer: A

Solution:

Using section formula

$C\left(\frac{-2a+b}{a+b}, \frac{a+2b}{a+b}\right)$

Now, substitute point C on line equation $(2x+y-3=0)$.

$\Rightarrow \quad 2\left(\frac{-2a+b}{a+b}\right)+\frac{a+2b}{a+b}=3$

On solving, we get

$b=6a$

On substituting $b=6a$ in point C, we get

$$\left(\frac{4}{7}, \frac{13}{7}\right)$$

$$\because P\left(\frac{b}{3a}, -3\right)=P(2,-3)$$

and $Q\left(-3, -\frac{b}{3a}\right)=Q(-3,-2)$

Now, coordinate of C is

$C\left(\frac{-3p+2q}{p+q}, \frac{-2p-3q}{p+q}\right)$

On comparing, we get

$\frac{-3p+2q}{p+q}=\frac{4}{7}$	$\frac{-2p-3q}{p+q}=\frac{13}{7}$
$\frac{p}{q}=\frac{2}{5}$	$\frac{p}{q}=\frac{-34}{27}$

Now, $\frac{p}{q}+\frac{q}{p}=\frac{2}{5}+\frac{5}{2}=\frac{4+25}{10}=\frac{29}{10}$

Question25

If Q and R are the images of the point $P(2,3)$ with respect to the lines $x-y+2=0$ and $2x+y-2=0$ respectively, then Q and R lie on

Options:

A. the same side of the line $2x + y - 2 = 0$

B. the opposite sides of the line $2x - y - 2 = 0$

C. the same side of the line $x + y + 2 = 0$

D. the opposite sides of the line $x - y + 2 = 0$

Answer: C

Solution:

Let $Q \equiv (a, b)$ and $R = (c, d)$

Then,

$\Rightarrow \frac{a-2}{1} = \frac{b-3}{-1} = \frac{-2(2-3+2)}{2}$

$\Rightarrow a - 2 = 3 - b = -(1)$

$\Rightarrow a - 2 = -1; 3 - b = -1 \Rightarrow a = 1, b = 4 \because \frac{c-2}{2} = \frac{d-3}{1} = \frac{-2(4+3-2)}{5}$

$\Rightarrow c - 2 = -4, d - 3 = -2$

$\Rightarrow \quad c = -2, d = 1$

Option (c)

$P(Q) = P(1, 4) = 1 + 4 + 2 = 7$ (positive)

$P(R) = P(-2, 1) = -2 + 1 + 2 = 1$ (positive)

Hence, point lie on same sides.

Question26

If $(2, -1)$ is the point of intersection of the pair of lines $2x^2 + axy + 3y^2 + bx + cy - 3 = 0$, then $3a + 2b + c =$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. 11

B. 0

C. 1

D. 21

Answer: A

Solution:

To find point of intersection we d_0 partial differentiation

On differentiating w.r.t. x, we get

$4x + ay + b = 0$

On differentiating w.r.t. y, we get

$ax + 6y + c = 0$

$ax + 6y + c = 0$

On putting $(2, -1)$ in Eqs. (i) and (ii), we get

$8 - a + b = 0 \Rightarrow a - b = 8$

$2a - 6 + c = 0 \Rightarrow 2a + c = 6$

Also,

$(2, -1)$ satisfies the given equation

$8 - 2a + 3 + 2b - c - 3 = 0$

$$-2a + 2b - c = -8$$

$$2a - 2b + c = 8$$

Put $2a + c = 6$ from Eqs. (iv) to (v)

$\Rightarrow \quad 6 - 2b = 8 \Rightarrow -2b = 2$

$b = -1 \Rightarrow \quad a = 7$

$\therefore \quad c = -8$

Hence, $3a + 2b + c = 21 - 2 - 8 = 11.$

Question27

If the ratio of the distances of a variable point P from the point $(1, 1)$ and the line $x - y + 2 = 0$ is $1 : \sqrt{2}$, then the equation of the locus of P is

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. $x^2 + 2xy + y^2 - 8x = 0$

B. $3x^2 + 2xy + 3y^2 - 12x - 4y + 4 = 0.$

C. $x^2 + 2xy + y^2 - 12x + 4y + 4 = 0$

D. $x^2 + 2xy + y^2 - 8x + 8y = 0$

Answer: B

Solution:

Let $P = (h, k)$

Now, according to the question,

$$\frac{\sqrt{(h-1)^2 + (k-1)^2}}{\left|\frac{h-k+2}{\sqrt{1^2+(-1)^2}}\right|} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{2\left[(h-1)^2 + (k-1)^2\right]}{(h-k+2)^2} = \frac{1}{2}$$

[squaring both sides]

$$\Rightarrow \quad 4\left[\left(h^2 - 2h + 1\right) + \left(k^2 - 2k + 1\right)\right]$$

$$= h^2 + k^2 + 4 - 2hk - 4k + 4h$$

$$\Rightarrow \quad 3h^2 + 2hk + 3k^2 - 12h - 4k + 4 = 0$$

$\therefore$ The equation of the locus of P is

$3x^2 + 2xy + 3y^2 - 12x - 4y + 4 = 0$

Question28

If the origin is shifted to the point $\left(\frac{3}{2}, -2\right)$ by the translation of axes, then the transformed equation of $2x^2 + 4xy + y^2 + 2x - 2y + 1 = 0$ is

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. $4x^2 + 8xy + 2y^2 - 16 = 0$

B. $2x^2 - 4xy + y^2 = 0$

C. $4x^2 + 8xy + 2y^2 + 9 = 0$

D. $2x^2 - 4xy + y^2 + 16 = 0$

Answer: C

Solution:

Interchanging x with $x + \frac{3}{2}$ and y with $y + (-2)$ i.e. $y - 2$ in the given equation to get the required equation.

$\therefore$ Required equation :

$$2\left(x + \frac{3}{2}\right)^2 + 4\left(x + \frac{3}{2}\right)(y - 2)$$

$$+(y-2)^2 + 2\left(x + \frac{3}{2}\right) - 2(y-2) + 1 = 0$$

$$\Rightarrow \frac{2(2x+3)^2}{4} + \frac{4(2x+3)}{2}(y-2)$$

$$+(y-2)^2 + \frac{2(2x+3)}{2} - 2(y-2) + 1 = 0$$

$$\Rightarrow \frac{1}{2}\left[4x^2 + 12x + 9 + (8x + 12)\right.$$

$$\left.(y-2) + 2(y-2)^2 + 4x + 6 - 4(y-2) + 2\right]$$

$$= 0$$

$\Rightarrow 4x^2 + 12x + 9 + 8xy + 12y$

$-16x - 24 + 2y^2 - 8y + 8 + 4x + 6$

$\Rightarrow 4y + 8 + 2 = 0$

$\Rightarrow 4x^2 + 8xy + 2y^2 + 9 = 0$

Question29

$L \equiv x\cos\alpha + y\sin\alpha - p = 0$ represents a line perpendicular to the line $x + y + 1 = 0$. If p is positive, α lies in the fourth quadrant and perpendicular distance from $(\sqrt{2}, \sqrt{2})$ to the line, $L = 0$ is 5 units, then $p =$

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. 5

B. $\frac{5}{2}$

C. 10

D. $\frac{15}{2}$

Answer: A

Solution:

Given,

$L \equiv x\cos\alpha + y\sin\alpha - p = 0$

L is perpendicular to the line

$x + y + 1 = 0$

p is positive, $\alpha \in (270^\circ, 360^\circ)$ and

Distance of point $(\sqrt{2}, \sqrt{2})$ to the line L is 5 units.

Slope of $L = \frac{-\cos\alpha}{\sin\alpha} = -\cot\alpha$

Slope of line $x + y + 1 = 0$ is -1

$\therefore \quad (-\cot\alpha)(-1) = -1 \Rightarrow \cot\alpha = -1$

Now, $L \equiv x\cos 315^\circ + y\sin 315^\circ - p = 0$

$$\Rightarrow \quad x\left(\frac{1}{\sqrt{2}}\right) + y\left(-\frac{1}{\sqrt{2}}\right) - p = 0$$

$$\Rightarrow \quad x - y - p\sqrt{2} = 0$$

According to the question,

$$\frac{|\sqrt{2}-\sqrt{2}-p\sqrt{2}|}{\sqrt{1^2+(-1)^2}} = 5$$

$$\Rightarrow \quad p\sqrt{2}\,| = 5\sqrt{2}$$

$$\Rightarrow \quad p = 5 \quad [\because p > 0]$$

Question30

$(-2, -1), (2, 5)$ are two vertices of a triangle and $\left(2, \frac{5}{3}\right)$ is its orthocenter. If (m, n) is the third vertex of that triangle, then $m + n$ is equal to.

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. -4

B. -2

C. 5

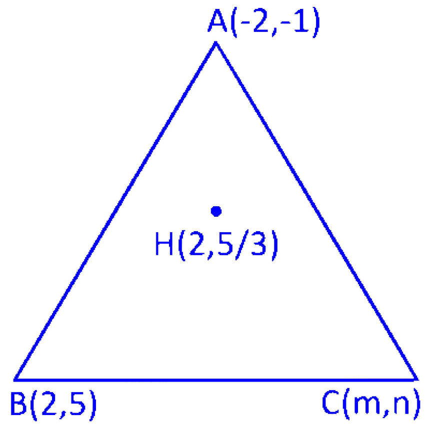
D. 8

Answer: C

Solution:

Given, $(-2,-1),(2,5)$ and (m,n) are three vertices of a triangle whose orthocenter is $\left(2, \frac{5}{3}\right)$.

Here, line $AH \perp$ line BC and line $CH \perp$ line AB



$$m_{AB}=\frac{5-(-1)}{2-(-2)}=\frac{6}{4}=\frac{3}{2}$$

$$m_{AH}=\frac{\frac{5}{3}-(-1)}{2-(-2)}=\frac{8}{12}=\frac{2}{3}$$

$$m_{BC}=\frac{n-5}{m-2}$$

$$\Rightarrow \quad m_{CH}=\frac{\frac{5}{3}-n}{2-m}=\frac{5-3n}{6-3m}$$

$$\Rightarrow \quad m_{AH}\times m_{BC}=-1$$

$$\frac{2}{3}\times\frac{n-5}{m-2}=-1$$

$$\Rightarrow \quad m_{CH}\times m_{AB}=-1 \quad \ldots \text{(i)}$$

$$\frac{5-3n}{6-3m}\times\frac{3}{2}=-1$$

On solving Eqs. (i) and (ii), we get $m=6$ and $n=-1$

$\therefore m+n=6+(-1)=5$

Question31

$L_1 \equiv 2x + y - 3 = 0$ and $L_2 \equiv ax + by + c = 0$ are two equal sides of an isosceles triangle. If $L_3 \equiv x + 2y + 1 = 0$ is the third side of this triangle and $(5, 1)$ is a point on $L_2 = 0$, then $\frac{b^2}{|ac|} =$

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. $\frac{121}{2}$

B. $\frac{49}{52}$

C. $\frac{81}{49}$

D. $\frac{25}{4}$

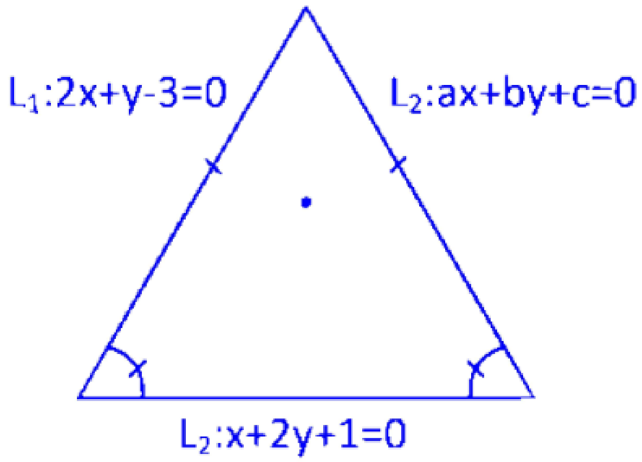
Answer: A

Solution:

Let the slopes of lines L_1, L_2 and L_3 be m_1, m_2 and m_3.

Here, $m_1 = -2$ and $m_3 = -\frac{1}{2}$

Now, angle between L_1 and L_3 will be equal to angle between L_2 and L_3.



$\therefore \quad \tan^{-1}\left|\frac{m_3-m_1}{1+m_1m_3}\right|$

$= \tan^{-1}\left|\frac{m_3-m_2}{1+m_2m_3}\right|$

$\Rightarrow \quad \left|\frac{-\frac{1}{2}+2}{1+(-2)(-\frac{1}{2})}\right| = \left|\frac{-\frac{1}{2}-m_2}{1-\frac{1}{2}m_2}\right|$

$$\Rightarrow \quad \left(\frac{-\frac{1}{2}-m_2}{1-\frac{1}{2}m_2}\right)^2 = \left(\frac{3}{4}\right)^2$$

$$\Rightarrow \quad m_2 = -2\frac{2}{11}$$

Discard $m_2 = -2$ because if $m_2 = -2$, then L_1 and L_2 will be parallel to each other.

$\therefore \quad m_2 = \frac{2}{11}$

The line L_2 is passing through the point ($5, \mathrm{f}$).

$\therefore \quad L_2 : (y-1) = \frac{2}{11}(x-5)$

$\Rightarrow \quad 2x - 11y + 1 = 0$

Hence, $a = 2b = -11$ and $c = 1$

So, $\frac{b^2}{|ac|} = \frac{(-11)^2}{|2\times1|} = \frac{121}{2}$

Question32

The slope of one of the pair of lines $2x^2 + hxy + 6y^2 = 0$ is thrice the slope of the other line, then $h =$

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. ±16

B. ±9

C. ±18

D. ±8

Answer: D

Solution:

Given pair of lines

$2x^2 + hxy + 6y^2 = 0$

Let the equations of lines be $y = mx$ and

$y = 3mx$.

Now, pair of equation of these lines will

be $(y - mx)(y - 3mx) = 0$

$\Rightarrow y^2 - mxy - 3mxy + 3m^2x^2 = 0$

$\Rightarrow 3m^2x^2 - 4mxy + y^2 = 0$

$\therefore \frac{3m^2}{2} = \frac{-4m}{h} = \frac{1}{6} \Rightarrow \frac{3m^2}{2} = \frac{1}{6}$

$\Rightarrow m^2 = \frac{1}{9}$ and $\frac{-4m}{h} = \frac{1}{6}$

$\Rightarrow h = -24m$

$\Rightarrow \quad h^2 = (24)^2m^2 = 576\left(\frac{1}{9}\right)$

$\Rightarrow \quad h^2 = 64$

$\Rightarrow h = \pm 8$

Question33

When the origin is shifted to the point $(2, b)$ by translation of axes, the coordinates of the point $(a, 4)$ have changed to $(6, 8)$. When the origin is shifted to (a, b) by translation of axes, if the transformed equation of $x^2 + 4xy + y^2 = 0$ is $X^2 + 2HXY + Y^2 + 2GX + 2FY + C = 0$, then $2H(G + F) =$

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. C

B. $-2C$

C. $2C$

D. -C

Answer: D

Solution:

When origin is shifted to $(2, b)$ by translation of axes, the coordinates of the point $(a, 4)$ have changed to $(6, 8)$

$\therefore X = x - 2 \Rightarrow Y = y - b$

For point $(a, 4)$ transforming to $(6, 8)$

$6 = a - 2 \Rightarrow a = 8$

$8 = 4 - b \Rightarrow b = -4$

So, the point (a, b) is $(8, -4)$.

Now, when origin is shifted to (a, b)

$\Rightarrow \quad X = x - 8 \Rightarrow Y = y + 4$

and transformed equation of $x^2 + 4xy + y^2 = 0$ is

$(X + 8)^2 + 4(X + 8)(Y - 4) + (Y - 4)^2 = 0$

$X^2 + Y^2 + 4XY + 24Y - 48 = 0$

On comparing with

$X^2 + 2HXY + Y^2 + 2GX + 2FY + C = 0$

we get, $G = 0, H = 2, F = 12, C = -48$

$\therefore 2H(G + F) = 2 \times 2(0 + 12) = 48 = -C$

Question34

The slope of a line L passing through the point $(-2, -3)$ is not defined. If the angle between the lines L and $ax - 2y + 3 = 0 (a > 0)$ is 45°, then the angle made by the line $x + ay - 4 \doteq 0$ with positive X-axis in the anti-clockwise direction is

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $\pi - \tan^{-1}\left(\frac{1}{2}\right)$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\tan^{-1}\left(\frac{1}{2}\right)$

Answer: A

Solution:

Since, slope of line L passing through $(-2,-3)$ is not defined $\Rightarrow \quad L_1 : x = -2$ and $m_1 = \pm\infty$ and given line is $ax - 2y + 3 = 0 (a > 0)$

$m_2 = \frac{a}{2}$

Given, $\tan 45^\circ = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$\tan 45^\circ = \left|\frac{\frac{a}{2} - \infty}{1 + \frac{a}{2} \cdot \infty}\right|$ or $\left|\frac{\frac{a}{2} + \infty}{1 - \frac{a}{2} \cdot \infty}\right|$

$\Rightarrow a = 2$ (considering the 45° angle with the vertical line)

Let θ be the angle made by the line $x + ay - 4 = 0$ with positive X-axis in the anti-clockwise direction.

$$\therefore \qquad \tan\theta = \frac{-1}{a} = \frac{-1}{2}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(-\frac{1}{2}\right) = \pi - \tan^{-1}\left(\frac{1}{2}\right)$$

Question35

(a, b) is the point of concurrency of the lines $x - 3y + 3 = 0$, $kx + y + k = 0$ and $2x + y - 8 = 0$. If the perpendicular distance from the origin to the line $L = ax - by + 2k = 0$ is p, then the perpendicular distance from the point $(2, 3)$ to $L = 0$ is

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $\frac{P}{2}$

B. p

C. $2p$

D. $3p$

Answer: B

Solution:

We have,

$x-3y+3=0 \quad \ldots\ldots.(i)$

$kx+y+k=0 \ \ldots\ldots.(ii)$

$2x+y-8=0 \ \ldots\ldots..(iii)$

On solving Eqs. (i) and (iii), we get

$x=3, y=2$

From Eq. (ii), we get

$3k+2+k=0 \Rightarrow K=-\frac{1}{2}$

($\because$ Eqs. (i), (ii) and (iii) are concurrent) and $a=3, b=2$ [point of concurrency is $(3,2)$]

So, $L \equiv 3x-2y-1=0$

given, distance from origin to L is P

$P=\frac{|0-0-1|}{\sqrt{3^2+(-2)^2}}=\frac{1}{\sqrt{13}}$

Hence, perpendicular distance from the point $(2,3)$ to L is

$=\frac{|6-6-1|}{\sqrt{3^2+(-2)^2}}=\frac{1}{\sqrt{13}}=P$

Question36

If $(4,3)$ and $(1,-2)$ are the end points of a diagonal of a square, then the equation of one of its sides is

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $4x+y-11=0$

B. $2x+y=0$

C. $2x-3y+1=0$

D. $x-4y-9=0$

Answer: D

Solution:

Slope of diagonal $= \frac{-2-3}{1-4} = \frac{5}{3}$

Let slope of a side of square be m.

$\because$ Angle between a diagonal and a side in square $= 45^\circ$

$\therefore \tan 45^\circ = \left|\frac{\frac{5}{3}-m}{1+\frac{5}{3}m}\right| \Rightarrow 1 = \left|\frac{\frac{5}{3}-m}{1+\frac{5}{3}m}\right|$

$\Rightarrow \quad m = \frac{1}{4}, -4$

Then, equation of one of the side of square is

$$y-(-2) = \frac{1}{4}(x-1) \Rightarrow 4(y+2) = x-1$$

$x - 4y - 9 = 0$

Question37

Area of the triangle bounded by the lines given by the equations $12x^2 - 20xy + 7y^2 = 0$ and $x + y - 1 = 0$ is

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $\frac{8}{29}$

B. $\frac{8}{39}$

C. $\frac{4}{29}$

D. $\frac{4}{39}$

Answer: D

Solution:

We have, $12x^2 - 20xy + 7y^2 = 0$

$(6x - 7y)(2x - y) = 0$

$\therefore$ Sides of triangle are

$6x - 7y = 0 \quad \ldots$ (i)

$2x - y = 0 \quad \ldots$ (ii)

$x + y = -1 \quad \ldots$ (iii)

From Eqs. (i), (ii) and (iii), we get the coordinates of vertices are

$(0,0), \left(-\frac{1}{3}, -\frac{2}{3}\right), \left(-\frac{7}{13}, -\frac{6}{13}\right)$

$$\text{So, area of triangle} = \left|\frac{1}{2}\right| \begin{matrix} 0 & 0 & 1 \\ -\frac{1}{3} & -\frac{2}{3} & 1 \\ -\frac{7}{13} & -\frac{6}{13} & 1 \end{matrix} \Bigg|$$

$$= \left|\frac{1}{2}\left(\frac{6}{39} - \frac{14}{39}\right)\right|$$

$$= \left|\frac{1}{2} \times \left(-\frac{8}{39}\right)\right| = \frac{4}{39}$$

Question38

A line L has intercepts a and b on the coordinate axes. When the coordinate axes are rotated through an angle α and keeping the origin fixed, the same line L has intercepts p and q on the new axes. Then,

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $a^2 + b^2 = p^2 + q^2$

B. $a^2 + p^2 = b^2 + q^2$

C. $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

D. $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

Answer: D

Solution:

Let us suppose that the axis are rotated in the anti-clockwise direction through an angle α. The equation of the line L with respect to the old axis is given by $\frac{x}{a}+\frac{y}{b}=1$.

Now, to find the equation of L with respect to the new axes, we replace ' x by $x \cos \alpha-y \sin \alpha$ and y by $x \sin \alpha+y \cos \alpha$ so that the equation of ' L ' with respect to the new axis is

$\frac{1}{a}(x \cos \alpha-y \sin \alpha)$

$+\frac{1}{b}(x \sin \alpha+y \cos \alpha)=1$

Since, the line ' L ' intercepts p and q on the new axes. So, we have on putting ($P, 0$) and then ($0, q$)

So, $\frac{1}{p}=\frac{1}{a} \cos \alpha+\frac{1}{b} \sin \alpha$ and $\frac{1}{q}=-\frac{1}{a} \sin \alpha+\frac{1}{b} \cos \alpha$

Now, eliminating ' α ', we get

$\frac{1}{p^2}+\frac{1}{q^2}=\frac{1}{a^2}+\frac{1}{b^2}$

Question39

Two lines L_1 and L_2 passing through the point $P(1,2)$ cut the line $x+y=4$ at a distance of $\frac{\sqrt{6}}{3}$ units from P. Then, the angles made by L_1, L_2 with positive X-axis are

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $\frac{\pi}{3}, \frac{\pi}{6}$

B. $\frac{\pi}{8}, \frac{3\pi}{8}$

C. $\frac{\pi}{12}, \frac{5\pi}{12}$

D. $\frac{\pi}{4}, \frac{\pi}{8}$

Answer: C

Solution:

Given point $P(1,2)$, we need to find the coordinates of points on lines L_1 and L_2 which intersect the line $x+y=4$ at a distance of $\frac{\sqrt{6}}{3}$ units from P.

Using the parametric formula for points at a given distance, the coordinates are:

$$\left(1+\sqrt{\tfrac{2}{3}}\cos\theta, 2+\sqrt{\tfrac{2}{3}}\sin\theta\right)$$

This point must satisfy the equation of the line $x+y=4$:

$$\Rightarrow\left(1+\sqrt{\frac{2}{3}}\cos\theta\right)+\left(2+\sqrt{\frac{2}{3}}\sin\theta\right)=4$$

$$\Rightarrow 3+\sqrt{\frac{2}{3}}(\cos\theta+\sin\theta)=4$$

$$\Rightarrow\sqrt{\frac{2}{3}}(\cos\theta+\sin\theta)=1$$

Squaring both sides:

$$\frac{2}{3}(1+\sin 2\theta)=1$$

$$\Rightarrow 1+\sin 2\theta=\frac{3}{2}$$

$$\Rightarrow\sin 2\theta=\frac{1}{2}$$

The solutions to $\sin 2\theta=\frac{1}{2}$ are:

$$\theta=\frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)$$

Which gives the angles:

$$\theta=\frac{\pi}{12} \quad \text{and} \quad \frac{5\pi}{12}$$

Thus, the angles made by lines L_1 and L_2 with the positive X-axis are $\frac{\pi}{12}$ and $\frac{5\pi}{12}$.

Question40

A pair of straight lines drawn though the origin forms. an isosceles triangle right angled at the origin with the line $2x+3y=6$. The area (in sq units) of the triangle, so formed is

Options:

A. $36/13$

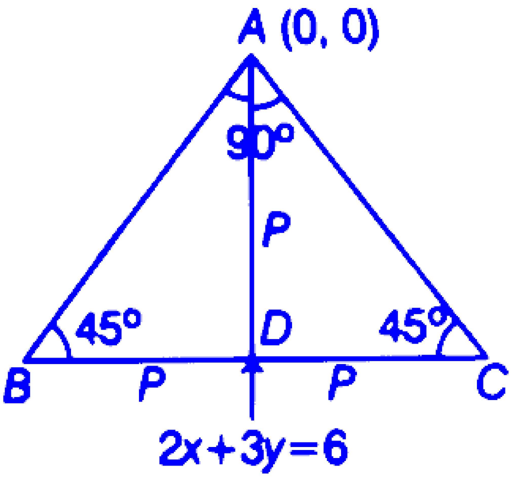
B. $32/13$

C. $28/9$

D. $26/9$

Answer: A

Solution:



Let $'L' = 2x + 3y = 6$

Any line through the origin making an angle of 45° with the given line $2x + 3y = 6$ is of the form $y = mx$, where

$$\tan(\pm 45^\circ) = \frac{m - \left(-\frac{2}{3}\right)}{1 + m\left(-\frac{2}{3}\right)}$$

$$\Rightarrow \pm 1 = \frac{3m + 2}{(3 - 2m)} \Rightarrow (3m + 2) = \pm(3 - 2m)$$

Either, $3m + 2 = 3 - 2m$

$\Rightarrow \quad m = \frac{1}{5}$

Or, $3m+2=-(3-2m)$

$\Rightarrow \quad m=-5$

Hence, the other two sides are $y=\frac{1}{5}x$ and $y=-5x$

i.e. $x-5y=0$ and $5x+y=0$

Let ' P ' be the perpendicular from $A(0,0)$ to the base $2x+3y=6$

So, $P=\left|\frac{2\cdot 0+3\cdot 0-6}{\sqrt{4+9}}\right|=\frac{6}{\sqrt{13}}$

Now, the required area is

$$
\begin{aligned}
& =\frac{1}{2}\times BC\times AD \\
& =\frac{1}{2}\times 2P\times P=P^2=\frac{36}{13} \text{ sq unit}
\end{aligned}
$$

Question41

The equation of the straight line passing through the point $(3,2)$ and inclined at an angle of 60° with the line $\sqrt{3}x+y=1$ is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $\sqrt{3}x+y-(2+3\sqrt{3})=0$

B. $\sqrt{3}x-y+(2-3\sqrt{3})=0$

C. $-\sqrt{3}x+y-(2-3\sqrt{3})=0$

D. $-\sqrt{3}x+y+(2-3\sqrt{3})=0$

Answer: B

Solution:

The given line is, $\sqrt{3}x+y=1$

So, slope of the line is $m_1=-\sqrt{3}$

Let, the slope of the line L which passing through the point $(3, 2)$ is m_2.

Now,

$$\tan 60^\circ = \left|\frac{m_2 + \sqrt{3}}{1 - \sqrt{3}m_2}\right| = \pm\sqrt{3}$$
$$\Rightarrow \left(m_2 + \sqrt{3}\right) = \pm\sqrt{3}\left(1 - \sqrt{3}m_2\right)$$
$$\Rightarrow m_2 = 0 \text{ and } m_2 = \sqrt{3}$$

Now, equation of the line L passing through $(3, 2)$ is

$$(y-2) = \sqrt{3}(x-3)$$
$$\Rightarrow y - 2 = \sqrt{3}x - 3\sqrt{3}$$
$$\Rightarrow \sqrt{3}x - y + (2 - 3\sqrt{3}) = 0$$

Question42

An equilateral triangle is constructed between the lines $\sqrt{3}x + y - 6 = 0$ and $\sqrt{3}x + y + 9 = 0$ with base on one line and vertex on the other. The area (in sq units) of the triangle, so formed is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $\frac{175}{6\sqrt{3}}$

B. $\frac{225}{2\sqrt{3}}$

C. $\frac{225}{4\sqrt{3}}$

D. $\frac{245}{4\sqrt{2}}$

Answer: C

Solution:

Given lines are

$$\sqrt{3}x + y - 6 = 0$$

$$\Rightarrow \quad y = -\sqrt{3}x + 6 \quad \ldots (i)$$

and $$\sqrt{3}x + y + 9 = 0$$

$$\Rightarrow \quad y = -\sqrt{3}x - 9 \quad \ldots (ii)$$

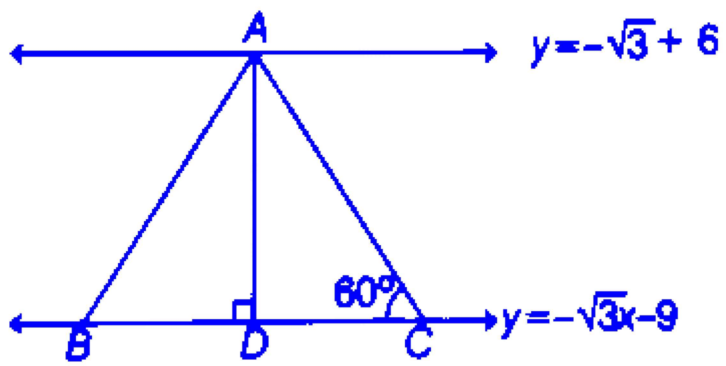
$\because$ Slope of both lines are same.

$\therefore$ These line are parallel.

So, perpendicular distance between both lines.

$= \frac{|c_1 - c_2|}{\sqrt{1+m^2}} = \frac{|6-(-9)|}{\sqrt{1+(-\sqrt{3})^2}} = \frac{15}{2}$

Then, equilateral triangle will be



$AD = \frac{15}{2}$ and $AB = BC = AC = a$

Then, $DC = \frac{BC}{2} = \frac{a}{2}$

In $\triangle ADC$, $\tan 60^\circ = \frac{AD}{DC} = \frac{(15/2)}{(a/2)}$

$\Rightarrow \quad a = \frac{15}{\sqrt{3}} = 5\sqrt{3}$

Therefore, Area ($\triangle ABC$)

$= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(5\sqrt{3})^2 = \frac{75\sqrt{3}}{4}(\text{ units })^2$

Question43

If θ is the acute angle between the lines joining the origin to the points of intersection of the curve $x^2 + xy + y^2 + x + 3y + 1 = 0$ and the straight line $x + y + 2 = 0$, then $\cos\theta =$

Options:

A. $\frac{1}{\sqrt{3}}$

B. $\frac{1}{\sqrt{5}}$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

Answer: B

Solution:

Equation of the line

$x+y+2=0 \qquad \ldots(i)$

and the equation of the curve is

$x^2+xy+y^2+x+3y+1=0 \qquad \ldots(ii)$

Now, from Eq. (i), we get $x+y+2=0$

$$\begin{aligned} &\Rightarrow x+y=-2 \\ &\Rightarrow \quad 1=\frac{-x-y}{2} \end{aligned}$$

From Eq. (ii), we get

$$\begin{aligned} x^2+xy+y^2+x+3y+1&=0 \\ \Rightarrow x^2+xy+y^2+x(1)+3y(1)+(1)^2&=0 \\ \Rightarrow x^2+xy+y^2+x\left(\frac{-x-y}{2}\right)+3y\left(\frac{-x-y}{2}\right)& \\ +\left(\frac{-x-y}{2}\right)^2&=0 \\ \Rightarrow x^2+xy+y^2+\left(\frac{-x^2-xy}{2}\right)+\left(\frac{-3xy-3y^2}{2}\right)& \\ +\left(\frac{x^2+y^2+2xy}{4}\right)&=0 \\ \Rightarrow 4x^2+4xy+4y^2-2x^2-2xy-6xy& \\ -6y^2+x^2+y^2+2xy&=0 \\ \Rightarrow 3x^2-y^2-2xy&=0 \\ \Rightarrow 3x^2-2xy-y^2&=0 \end{aligned}$$

Now, comparing this equation with $ax^2 + 2hxy + by^2 = 0$

Thus, $a = 3, h = -1, b = -1$

Now, θ be the acute angle (given)

So, $\cos\theta = \frac{|a+b|}{\sqrt{(a-b)^2+4h^2}}$

$= \frac{|3-1|}{\sqrt{(4)^2+4(-1)^2}} = \frac{2}{\sqrt{20}} = \frac{1}{\sqrt{5}}$

Question44

The angle, by which the coordinate axes are to be rotated about the origin so that the transformed equation of $\sqrt{3}x^2 + (\sqrt{3}-1)xy - y^2 = 0$ would be free from xy-term is

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. 45°

B. 22.5°

C. 15°

D. 7.5°

Answer: D

Solution:

Let axes be rotated through an angle θ, then old coordinates are

$$x = x\cos\theta - y\sin\theta$$
$$y = x\sin\theta + y\cos\theta$$
$$\therefore \sqrt{3}(x\cos\theta - y\sin\theta)^2$$
$$+(\sqrt{3}-1)(x\cos\theta - y\sin\theta)$$
$$(x\sin\theta + y\cos\theta) - (x\sin\theta + y\cos\theta)^2$$

Coefficient of $xy = 0$

$$\Rightarrow -2\sqrt{3}\cos\theta\cdot\sin\theta + (\sqrt{3}-1)$$
$$\left(\cos^2\theta - \sin^2\theta\right) - 2\sin\theta\cdot\cos\theta = 0$$
$$\Rightarrow -\sqrt{3}\sin 2\theta + (\sqrt{3}-1)\cos 2\theta - \sin 2\theta = 0$$

$\Rightarrow (\sqrt{3}-1)\cos 2\theta - (\sqrt{3}+1)\sin 2\theta = 0$

$\Rightarrow \tan 2\theta = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$

$\Rightarrow 2\theta = 15^\circ \Rightarrow \theta = 7.5^\circ$

Question45

If the slope of a straight line passing through $A(3,2)$ is $3/4$, then the coordinates of the two points on the same line that are 5 units away from A are

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. $(-7,5),(1,-1)$

B. $(7,5),(-1,-1)$

C. $(6,9),(-2,3)$

D. $(6,3),(-2,-3)$

Answer: B

Solution:

To find the coordinates of the points that are 5 units away from $A(3,2)$ on a line with a slope of $\frac{3}{4}$, we use the angle θ where $\tan\theta = \frac{3}{4}$.

First, we find the trigonometric values:

$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

Using the distance formula along the direction of the line, we set up the equations as follows:

$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = \pm r$

Substitute the known values:

$\frac{x-3}{\frac{4}{5}} = \pm 5$

$\frac{y-2}{\frac{3}{5}} = \pm 5$

Solve for x and y:

$x - 3 = \pm 4$, so $x = 3 \pm 4$.

$y - 2 = \pm 3$, so $y = 2 \pm 3$.

This results in the coordinates:

$A_1 = (3 + 4, 2 + 3) = (7, 5)$

$A_2 = (3 - 4, 2 - 3) = (-1, -1)$

So, the points on the line that are 5 units away from $A(3, 2)$ are $(7, 5)$ and $(-1, -1)$.

Question46

If each of the points $(a, 4)$, $(-2, b)$ lies on the line joining the points $(2, -1)$ and $(5, -3)$, then the point (a, b) lies on the line

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. $6x + 6y - 25 = 0$

B. $x + 3y + 1 = 0$

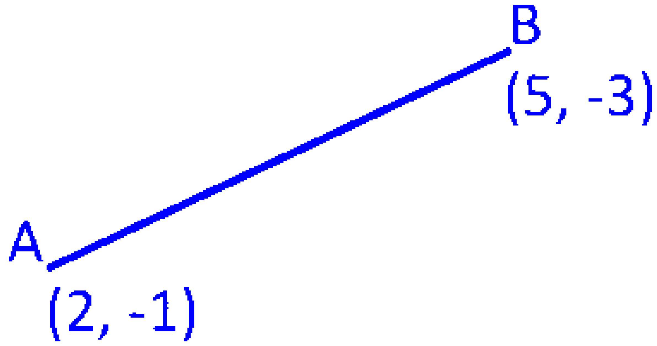
C. $2x + 6y + 1 = 0$

D. $2x + 3y - 5 = 0$

Answer: C

Solution:

Equation of AB



$y+1=\frac{-3+1}{5-2}(x-2)$

$$\Rightarrow \quad y+1=-\frac{2}{3}(x-2)$$
$$\Rightarrow \quad 3y+3=-2x+4$$
$$\Rightarrow \quad 2x+3y=1$$

$\because (a, 4)$ and $(-2, b)$ lies on AB

$\Rightarrow 2a+12=1$ and $-4+3b=1$

$$a=-\frac{11}{2}, b=\frac{5}{3}$$

(a,b) i.e. $\left(-\frac{11}{2}, \frac{5}{3}\right)$ lies on $2x+6y+1=0$
